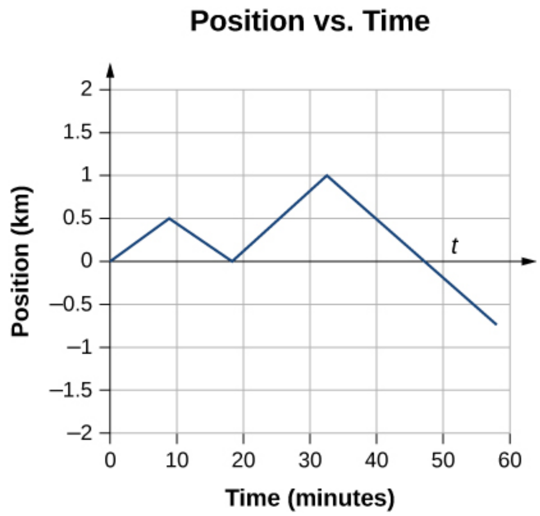
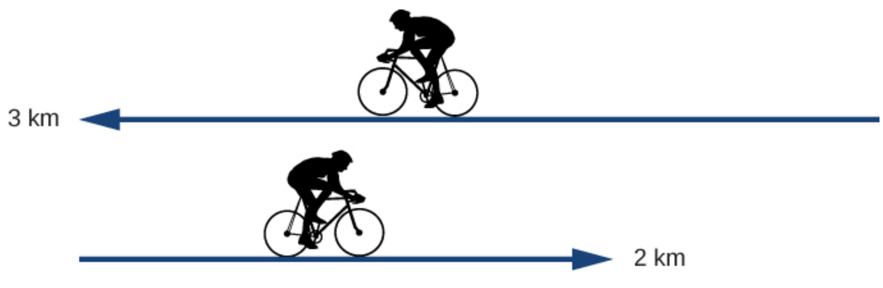


Figure 3.5 This graph depicts Jill's position versus time. The average velocity is the slope of a line connecting the initial and final points.

Significance

Jill's total displacement is −0.75 km, which means at the end of her trip she ends up $0.75\,\text{km}$ due west of her home. The average velocity means if someone was to walk due west at $0.013\ \text{km/min}$ starting at the same time Jill left her home, they both would arrive at the final stopping point at the same time. Note that if Jill were to end her trip at her house, her total displacement would be zero, as well as her average velocity. The total distance traveled during the 58 minutes of elapsed time for her trip is 3.75 km.

3.1 Check Your Understanding A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is his displacement? (b) What is the distance traveled? (c) What is the magnitude of his displacement?



3.2 | Instantaneous Velocity and Speed

Learning Objectives

By the end of this section, you will be able to:

- Explain the difference between average velocity and instantaneous velocity.
- Describe the difference between velocity and speed.
- Calculate the instantaneous velocity given the mathematical equation for the velocity.
- Calculate the speed given the instantaneous velocity.

We have now seen how to calculate the average velocity between two positions. However, since objects in the real world move continuously through space and time, we would like to find the velocity of an object at any single point. We can find the velocity of the object anywhere along its path by using some fundamental principles of calculus. This section gives us better insight into the physics of motion and will be useful in later chapters.

Instantaneous Velocity

The quantity that tells us how fast an object is moving anywhere along its path is the **instantaneous velocity**, usually called simply *velocity*. It is the average velocity between two points on the path in the limit that the time (and therefore the displacement) between the two points approaches zero. To illustrate this idea mathematically, we need to express position x as a continuous function of t denoted by $x(t)$. The expression for the average velocity between two points using this notation is $\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$. To find the instantaneous velocity at any position, we let $t_1 = t$ and $t_2 = t + \Delta t$. After inserting these expressions into the equation for the average velocity and taking the limit as $\Delta t \to 0$, we find the expression for the instantaneous velocity:

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = \frac{dx(t)}{dt}.$$

Instantaneous Velocity

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t:

$$v(t) = \frac{d}{dt}x(t). \tag{3.4}$$

Like average velocity, instantaneous velocity is a vector with dimension of length per time. The instantaneous velocity at a specific time point t_0 is the rate of change of the position function, which is the slope of the position function $x(t)$ at t_0. **Figure 3.6** shows how the average velocity $\bar{v} = \frac{\Delta x}{\Delta t}$ between two times approaches the instantaneous velocity at t_0. The instantaneous velocity is shown at time t_0, which happens to be at the maximum of the position function. The slope of the position graph is zero at this point, and thus the instantaneous velocity is zero. At other times, t_1, t_2, and so on, the instantaneous velocity is not zero because the slope of the position graph would be positive or negative. If the position function had a minimum, the slope of the position graph would also be zero, giving an instantaneous velocity of zero there as well. Thus, the zeros of the velocity function give the minimum and maximum of the position function.

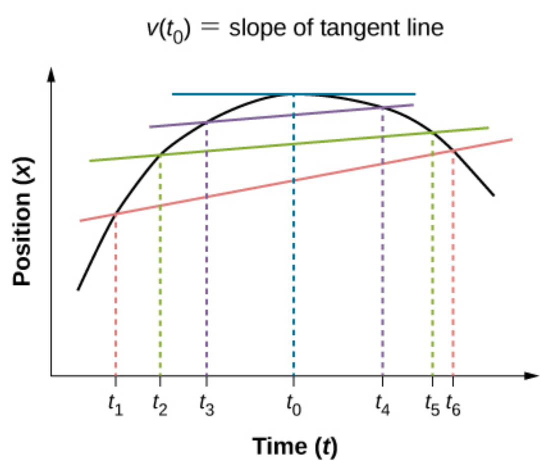


Figure 3.6 In a graph of position versus time, the instantaneous velocity is the slope of the tangent line at a given point. The average velocities $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ between times $\Delta t = t_6 - t_1$, $\Delta t = t_5 - t_2$, and $\Delta t = t_4 - t_3$ are shown. When $\Delta t \to 0$, the average velocity approaches the instantaneous velocity at $t = t_0$.

Example 3.2

Finding Velocity from a Position-Versus-Time Graph

Given the position-versus-time graph of Figure 3.7, find the velocity-versus-time graph.

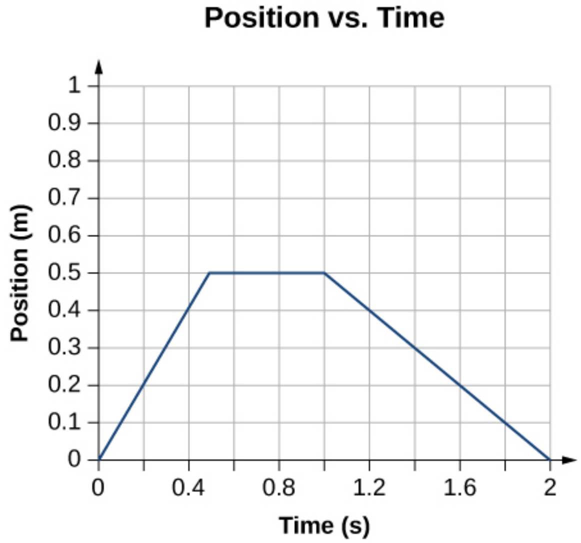


Figure 3.7 The object starts out in the positive direction, stops for a short time, and then reverses direction, heading back toward the origin. Notice that the object comes to rest instantaneously, which would require an infinite force. Thus, the graph is an approximation of motion in the real world. (The concept of force is discussed in Newton's Laws of Motion.)

Strategy

The graph contains three straight lines during three time intervals. We find the velocity during each time interval by taking the slope of the line using the grid.

Solution

Time interval 0 s to 0.5 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.5\text{ m} - 0.0\text{ m}}{0.5\text{ s} - 0.0\text{ s}} = 1.0\text{ m/s}$

Time interval 0.5 s to 1.0 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.5\text{ m} - 0.5\text{ m}}{1.0\text{ s} - 0.5\text{ s}} = 0.0\text{ m/s}$

Time interval 1.0 s to 2.0 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.0\text{ m} - 0.5\text{ m}}{2.0\text{ s} - 1.0\text{ s}} = -0.5\text{ m/s}$

The graph of these values of velocity versus time is shown in Figure 3.8.

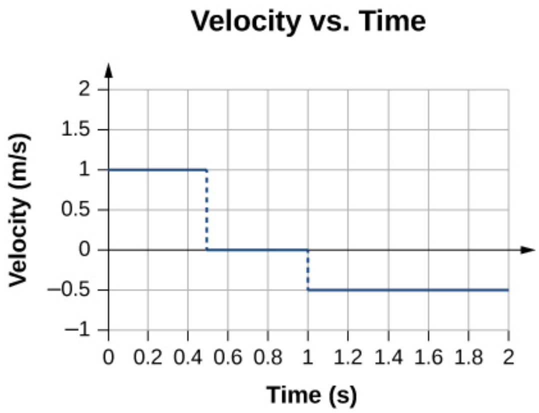


Figure 3.8 The velocity is positive for the first part of the trip, zero when the object is stopped, and negative when the object reverses direction.

Significance

During the time interval between 0 s and 0.5 s, the object's position is moving away from the origin and the position-versus-time curve has a positive slope. At any point along the curve during this time interval, we can find the instantaneous velocity by taking its slope, which is +1 m/s, as shown in **Figure 3.8**. In the subsequent time interval, between 0.5 s and 1.0 s, the position doesn't change and we see the slope is zero. From 1.0 s to 2.0 s, the object is moving back toward the origin and the slope is −0.5 m/s. The object has reversed direction and has a negative velocity.

Speed

In everyday language, most people use the terms *speed* and *velocity* interchangeably. In physics, however, they do not have the same meaning and are distinct concepts. One major difference is that speed has no direction; that is, speed is a scalar.

We can calculate the **average speed** by finding the total distance traveled divided by the elapsed time:

$$\text{Average speed} = \bar{s} = \frac{\text{Total distance}}{\text{Elapsed time}}. \tag{3.5}$$

Average speed is not necessarily the same as the magnitude of the average velocity, which is found by dividing the magnitude of the total displacement by the elapsed time. For example, if a trip starts and ends at the same location, the total displacement is zero, and therefore the average velocity is zero. The average speed, however, is not zero, because the total distance traveled is greater than zero. If we take a road trip of 300 km and need to be at our destination at a certain time, then we would be interested in our average speed.

However, we can calculate the **instantaneous speed** from the magnitude of the instantaneous velocity:

$$\text{Instantaneous speed} = |v(t)|. \tag{3.6}$$

If a particle is moving along the x-axis at +7.0 m/s and another particle is moving along the same axis at −7.0 m/s, they have different velocities, but both have the same speed of 7.0 m/s. Some typical speeds are shown in the following table.

Speed	m/s	mi/h
Continental drift	10^{-7}	2×10^{-7}
Brisk walk	1.7	3.9
Cyclist	4.4	10
Sprint runner	12.2	27
Rural speed limit	24.6	56
Official land speed record	341.1	763
Speed of sound at sea level	343	768
Space shuttle on reentry	7800	17,500
Escape velocity of Earth*	11,200	25,000
Orbital speed of Earth around the Sun	29,783	66,623
Speed of light in a vacuum	299,792,458	670,616,629

Table 3.1 Speeds of Various Objects *Escape velocity is the velocity at which an object must be launched so that it overcomes Earth's gravity and is not pulled back toward Earth.

Calculating Instantaneous Velocity

When calculating instantaneous velocity, we need to specify the explicit form of the position function $x(t)$. For the moment, let's use polynomials $x(t) = At^n$, because they are easily differentiated using the power rule of calculus:

$$\frac{dx(t)}{dt} = nAt^{n-1}. \tag{3.7}$$

The following example illustrates the use of **Equation 3.7**.

Example 3.3

Instantaneous Velocity Versus Average Velocity

The position of a particle is given by $x(t) = 3.0t + 0.5t^3$ m.

a. Using **Equation 3.4** and **Equation 3.7**, find the instantaneous velocity at $t = 2.0$ s.

b. Calculate the average velocity between 1.0 s and 3.0 s.

Strategy

Equation 3.4 gives the instantaneous velocity of the particle as the derivative of the position function. Looking at the form of the position function given, we see that it is a polynomial in t. Therefore, we can use **Equation 3.7**, the power rule from calculus, to find the solution. We use **Equation 3.6** to calculate the average velocity of the particle.

Solution

a. $v(t) = \dfrac{dx(t)}{dt} = 3.0 + 1.5t^2$ m/s.

Substituting t = 2.0 s into this equation gives $v(2.0\text{ s}) = [3.0 + 1.5(2.0)^2]\text{ m/s} = 9.0\text{ m/s}$.

b. To determine the average velocity of the particle between 1.0 s and 3.0 s, we calculate the values of x(1.0 s) and x(3.0 s):

$$x(1.0\ \text{s}) = \left[(3.0)(1.0) + 0.5(1.0)^3\right]\text{m} = 3.5\ \text{m}$$

$$x(3.0\ \text{s}) = \left[(3.0)(3.0) + 0.5(3.0)^3\right]\text{m} = 22.5\ \text{m}.$$

Then the average velocity is

$$\bar{v} = \frac{x(3.0\ \text{s}) - x(1.0\ \text{s})}{t(3.0\ \text{s}) - t(1.0\ \text{s})} = \frac{22.5 - 3.5\ \text{m}}{3.0 - 1.0\ \text{s}} = 9.5\ \text{m/s}.$$

Significance

In the limit that the time interval used to calculate $\bar{v}$ goes to zero, the value obtained for $\bar{v}$ converges to the value of *v*.

Example 3.4

Instantaneous Velocity Versus Speed

Consider the motion of a particle in which the position is $x(t) = 3.0t - 3t^2\ \text{m}$.

a. What is the instantaneous velocity at $t = 0.25$ s, $t = 0.50$ s, and $t = 1.0$ s?

b. What is the speed of the particle at these times?

Strategy

The instantaneous velocity is the derivative of the position function and the speed is the magnitude of the instantaneous velocity. We use **Equation 3.4** and **Equation 3.7** to solve for instantaneous velocity.

Solution

a. $v(t) = \dfrac{dx(t)}{dt} = 3.0 - 6.0t\ \text{m/s} \quad v(0.25\ \text{s}) = 1.50\ \text{m/s},\ v(0.5\ \text{s}) = 0\ \text{m/s},\ v(1.0\ \text{s}) = -3.0\ \text{m/s}$

b. $\text{Speed} = |v(t)| = 1.50\ \text{m/s},\ 0.0\ \text{m/s, and}\ 3.0\ \text{m/s}$

Significance

The velocity of the particle gives us direction information, indicating the particle is moving to the left (west) or right (east). The speed gives the magnitude of the velocity. By graphing the position, velocity, and speed as functions of time, we can understand these concepts visually **Figure 3.9**. In (a), the graph shows the particle moving in the positive direction until $t = 0.5$ s, when it reverses direction. The reversal of direction can also be seen in (b) at 0.5 s where the velocity is zero and then turns negative. At 1.0 s it is back at the origin where it started. The particle's velocity at 1.0 s in (b) is negative, because it is traveling in the negative direction. But in (c), however, its speed is positive and remains positive throughout the travel time. We can also interpret velocity as the slope of the position-versus-time graph. The slope of $x(t)$ is decreasing toward zero, becoming zero at 0.5 s and increasingly negative thereafter. This analysis of comparing the graphs of position, velocity, and speed helps catch errors in calculations. The graphs must be consistent with each other and help interpret the calculations.

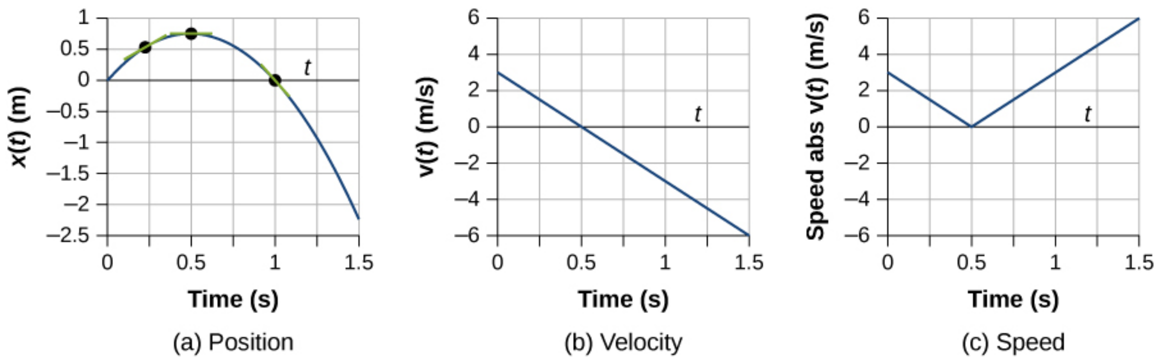


Figure 3.9 (a) Position: $x(t)$ versus time. (b) Velocity: $v(t)$ versus time. The slope of the position graph is the velocity. A rough comparison of the slopes of the tangent lines in (a) at 0.25 s, 0.5 s, and 1.0 s with the values for velocity at the corresponding times indicates they are the same values. (c) Speed: $|v(t)|$ versus time. Speed is always a positive number.

3.2 Check Your Understanding The position of an object as a function of time is $x(t) = -3t^2$ m . (a) What is the velocity of the object as a function of time? (b) Is the velocity ever positive? (c) What are the velocity and speed at t = 1.0 s?

3.3 | Average and Instantaneous Acceleration

Learning Objectives

By the end of this section, you will be able to:

- Calculate the average acceleration between two points in time.
- Calculate the instantaneous acceleration given the functional form of velocity.
- Explain the vector nature of instantaneous acceleration and velocity.
- Explain the difference between average acceleration and instantaneous acceleration.
- Find instantaneous acceleration at a specified time on a graph of velocity versus time.

The importance of understanding acceleration spans our day-to-day experience, as well as the vast reaches of outer space and the tiny world of subatomic physics. In everyday conversation, to *accelerate* means to speed up; applying the brake pedal causes a vehicle to slow down. We are familiar with the acceleration of our car, for example. The greater the acceleration, the greater the change in velocity over a given time. Acceleration is widely seen in experimental physics. In linear particle accelerator experiments, for example, subatomic particles are accelerated to very high velocities in collision experiments, which tell us information about the structure of the subatomic world as well as the origin of the universe. In space, cosmic rays are subatomic particles that have been accelerated to very high energies in supernovas (exploding massive stars) and active galactic nuclei. It is important to understand the processes that accelerate cosmic rays because these rays contain highly penetrating radiation that can damage electronics flown on spacecraft, for example.

Average Acceleration

The formal definition of acceleration is consistent with these notions just described, but is more inclusive.

Average Acceleration

Average acceleration is the rate at which velocity changes:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}, \tag{3.8}$$